Letters

Design of Coupled Microstrip Lines

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***Abstract*—An accurate and direct method for designing coupled microstrip lines for any substrate is given once the design curves for any other substrate are known.**

The design of circuits involving coupled microstrip lines needs determination of Z_{oe} and Z_{oo}, the even and odd mode impedances, respectively. The geometry of a coupled line is shown in Fig. 1. For an accurate determination of Z_{oe} and Z_{oo} in terms of the line parameters (W/H and S/H, see Fig. 1), one has to use computer programs [1] or design curves available in the literature [2]. But the design curves are given only for some specific substrate materials. In this letter we point out that if the design curves for one substrate material are known, they can easily be derived for other materials.

It turns out that the even and odd mode impedances for two substrate materials are approximately related by

$$\frac{Z_{oi1}}{Z_{oi2}} \simeq \left(\frac{\varepsilon_{\text{reff2}}}{\varepsilon_{\text{reff1}}}\right)^{1/2} \tag{1}$$

where $i = o$ or e. $\varepsilon_{\text{reff}}$ is the effective dielectric constant for a substrate, and it is given by [3]

$$\varepsilon_{\text{reff}} = \left(\frac{\varepsilon_r + 1}{2}\right) + \left(\frac{\varepsilon_r - 1}{2}\right)\left(1 + 10\,\frac{H}{W}\right)^{-1/2}. \tag{2}$$

In (2) ε_r is the relative permittivity of the substrate material. The subscripts 1 and 2 on $\varepsilon_{\text{reff}}$ refer to the substrate materials 1 and 2, respectively. Equation (2) is accurate within 2 percent for $0 < W/h < \infty$ and $1 < \varepsilon_r < \infty$ [3].

The physical basis, on which (1) is written down, is the fact that the characteristic impedances for transmission lines scale as $\varepsilon_r^{-1/2}$. Moreover, a microstrip line involves an inhomogeneous medium. Hence its impedances should scale as $\varepsilon_{\text{reff}}^{-1/2}$.

To check the accuracy of (1), two examples are taken. In one example $\varepsilon_{r1} = 9.6$ and $\varepsilon_{r2} = 3.7$ and in the other example $\varepsilon_{r1} = 9$ and $\varepsilon_{r2} = 6$. For these values of ε_r, design curves are available in the literature [2], [4], and [5]. In Table I we give the values of Z_{oe} and Z_{oo} for $\varepsilon_r = 3.7$ derived from (1) and the design curves for $\varepsilon_r = 9.6$. These derived values of Z_{oe} and Z_{oo} are compared with the values obtained from the design curves [4] available for $\varepsilon_r = 3.7$. In Table II values of Z_{oe} and Z_{oo} derived from (1) and the design curves for $\varepsilon_r = 9$ are compared with those obtained from the design curves [2] for $\varepsilon_r = 6$.

From Tables I and II we observe that the percentage errors in the derived values of the even and odd impedances are quite small. Thus they can be used for a first-order design of circuits involving coupled microstrip lines. The percentage error increases for small spacings S/H. When S/H is 0.05, we find that the error is about ± 3 percent. However, the spacings S/H less than 0.05 are difficult to realize in practice due to dimensional tolerances. Formulas (1) and (2) provide an easy and accurate method for the design of coupled microstrip lines for any desired substrate once design curves for any other substrate are available. However, this fact has not been appreciated in the literature.

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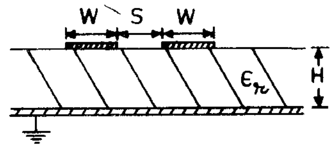

Fig. 1. Geometry of a coupled microstrip line.

TABLE I
TRANSFORMATION FROM $\varepsilon_{r1} = 9.6$ TO $\varepsilon_{r2} = 3.7$

(a) For Z_{oe}

W/H	S/H	Z_{oe1} (exact from Ref.4) Ohms	Z_{oe2} (approximate from eq.(1)) Ohms	Z_{oe2} (exact from ref.4) Ohms	Percentage error
	0.1	105	161	161.5	0.3
0.4	1.0	84	128.7	129.0	0.2
	5.0	74	113.4	113.0	0.3
	0.1	53	82.3	83.0	0.8
1.4	1.0	46.5	72.2	73.0	1.0
	5.0	42.5	66.0	65.5	0.8

(b) For Z_{oo}

W/H	S/H	Z_{oo1} (Exact from Ref.4) Ohms	Z_{oo2} (approximate from eq.(1)) Ohms	Z_{oo2} (exact from ref.4) Ohms	Percentage error
	0.1	36.0	55.2	53.5	2.8
0.4	1.0	62.0	95.0	94.0	1.2
	5.0	72.5	111.1	110.0	1.0
	0.1	25.0	38.8	38.0	2.5
1.4	1.0	36.5	56.7	55.5	2.5
	5.0	41.0	63.7	63.5	0.2

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TABLE II
TRANSFORMATION FROM $\varepsilon_{r1} = 9$ TO $\varepsilon_{r2} = 6$

(a) For Z_{oe}

W/H	S/H	Z_{oe1} (exact from ref.2) Ohms	Z_{oe2} (approximate from eq.(1)) Ohms	Z_{oe2} (exact from ref.2) Ohms	Percentage error
	0.05	110	132.4	134	1.1
0.4	0.5	95	114.4	115	0.5
	2.0	80	96.3	98	1.7
	0.05	56	67.9	68	0.1
1.4	0.5	52	63.1	63	0.1
	2.0	46	55.8	56	0.2

(b) For Z_{oo}

W/H	S/H	Z_{oo1} (exact from ref.2) Ohms	Z_{oo2} (approximate from eq.(1)) Ohms	Z_{oo2} (exact from ref.2) Ohms	Percentage error
	0.05	33	39.7	39	2.0
0.4	0.5	56	67.4	67	0.6
	2.0	71	85.5	85	0.6
	0.05	23	27.7	28	1.0
1.4	0.5	34	40.9	41	0.3
	2.0	41	49.4	49	1.0

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The Resonant Frequency of a Narrow-Gap Cylindrical Cavity

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***Abstract*—A recently proposed method for computing the resonant frequency of a narrow-gap reentrant cylindrical cavity is discussed. It is shown that provided that the cavity does not have too low a height-to-diameter ratio, its resonant frequency may also be computed with expectation of reasonable accuracy from numerical data which have been available in the literature for some time.**

INTRODUCTION

In a recent paper Williamson [1] has proposed a new method for computing the resonant frequency of the reentrant narrow-gap cylindrical cavity shown in Fig. 1, which is claimed to be simple and reasonably accurate. In the case of very squat cavities (low $h/2b$ ratio) it would seem that Williamson's method is a valuable contribution. However, when this is not the case, other methods, also fairly simple and potentially capable of good accuracy, are also possible.

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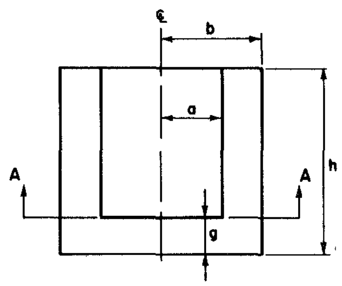

Fig. 1. Cross section of the reentrant cylindrical cavity.

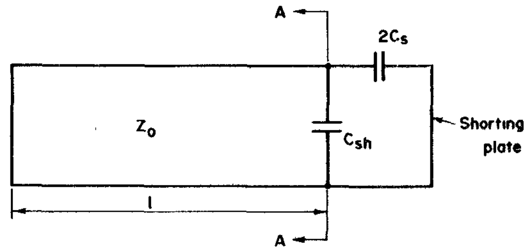

Fig. 2. Equivalent circuit of resonator.

THEORY

A gap in the inner conductor of a coaxial line may be modeled with good accuracy by an equivalent symmetrical π network of capacitors, provided that the gap is small compared with the wavelength in the line. Furthermore, under this condition, the values of the capacitances in the network can be computed from a quasi-static approximation. This has been done by Green [2], [3].

When a short-circuiting plane is introduced through the middle of the gap, the π network is bisected and the equivalent circuit of the resonator, the resonant frequency of which we wish to determine, is as given in Fig. 2. In this figure Z_o is the characteristic impedance of a coaxial line having inner conductor radius a and outer conductor radius b. C_s and C_{sh} are the series and shunt capacitances of the equivalent π network corresponding to a gap width of $2g$. They may be found from [2, table VI] by entering it with a gap ratio g/b and a diameter ratio b/a, finally multiplying the results extracted by $2\pi b$. In addition to the restrictions already cited, the value obtained for the gap capacitance will be valid only when the evanescent fields associated with the gap cannot interact with the short circuit. In practice, this means that the cavity should be so proportioned that $(h - g) > 2(b - a)$, and hence excludes some of the very squat cavities treated by Williamson.

At resonance the admittance seen at the plane AA must be zero. Hence the condition for resonance is

$$\cot \frac{\omega_r l}{c} - Z_o C_{eq} \omega_r = 0 \qquad (1)$$

where

ω_r resonant angular frequency;
l $= h - g$;
c velocity of propagation in the coaxial line (3×10^8 m/s);
C_{eq} $= 2C_s + C_{sh}$.